Geography (Hons.)
Sem III
Paper SEC-1

NUMBER SYSTEMS: BINARY ARITHMETIC

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What is a number system?

- A **numeral system** (or **system of numeration**) is a writing system for expressing numbers; that is, a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner.

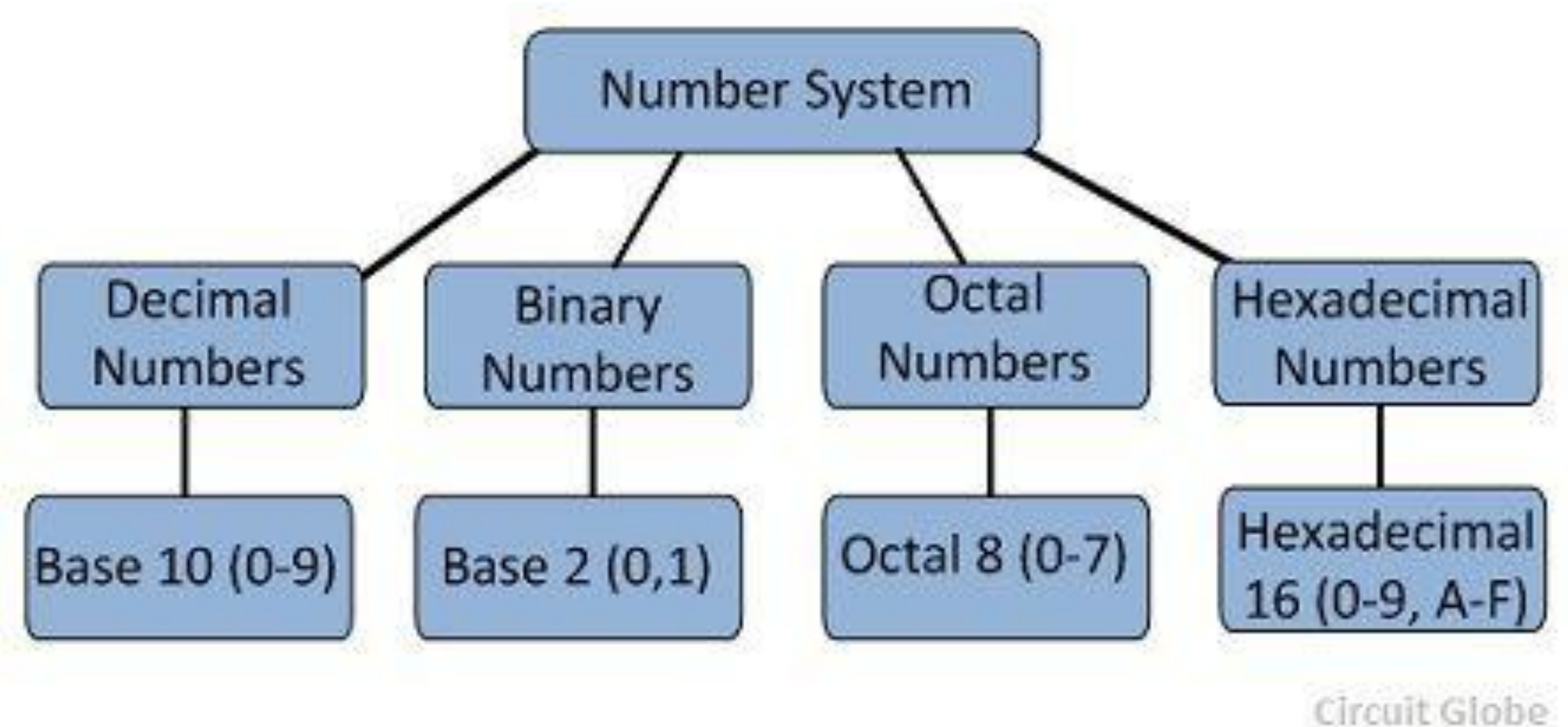
- The same sequence of symbols may represent different numbers in different numeral systems. For example, "11" represents the number *eleven* in the decimal numeral system (used in common life), the number *three* in the binary numeral system (used in computers), and the number two in the unary numeral system (e.g. used in tallying scores).

Characteristics

Ideally, a numeral system will:

- Represent a useful set of numbers (e.g. all integers, or rational numbers)
- Give every number represented a unique representation (or at least a standard representation)
- Reflect the algebraic and arithmetic structure of the numbers.

CLASSIFICATION OF NUMBER SYSTEMS



- All type of data processed/stored in a computer as binary numbers consisting of binary digits (bit).
- The value that may be taken by a bit is either 0 or 1, and can represent only two states – OFF and ON.
- For example, we might have 1-bit (or memory cell) that represents the number 0 when it is off and the number 1 when it is on.

Decimal Number System

- The decimal numbers are represented by arranging the symbols 0,1,2,3,4,5,6,7,8,9 also called decimal digits in various sequences.
- The position of each digit in a sequence has a certain numerical weight, and each digit is a multiplier of the weight of its position.
- The decimal number system is therefore an example of a weighted, positional number system.
- The weight of each position is a power of the base number 10.
- The value of a number is the sum of the products obtained by multiplying each digit by the weight of its respective position.

Example

- The number 345 (Fixed Number)

  $3x10^2 + 4x10^1 + 5x10^0 = 300 + 40 + 5 = 345$
- The number 123.45 (Floating Number)

  $1x10^2 + 2x10^1 + 3x10^0 + 4x10^{-1} + 5x10^{-2}$

  $= 100 + 20 + 3 + 0.4 + 0.05 = 123.45$

  The leftmost digit in any number representation, which has the greatest weight, is called the ***most significant digit (MSD),*** and the rightmost digit, which has the least weight, is called the ***least significant digit (LSD).***

Binary Number System

- A binary digit is called a ***bit***.
- A binary number consists of a sequence of bits.
- There are only two digits in binary number system *i.e.*, 0 and 1.
- The weight of each bit position is one power of 2 greater than the weight of the position to its right.
- The value of a binary number is the sum of all its bits multiplied by the weights of their respective positions.

Binary to Decimal Conversion

- $1010 = 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0 = 8+0+2+0 = 10$

  $(1010)_2 = (10)_{10}$

- $1101 = 1x2^3 + 1x2^2 + 0x2^1 + 1x2^0 = 8+4+0+1 = 13$

  $(1101)_2 = (13)_{10}$

Binary to Decimal Conversion

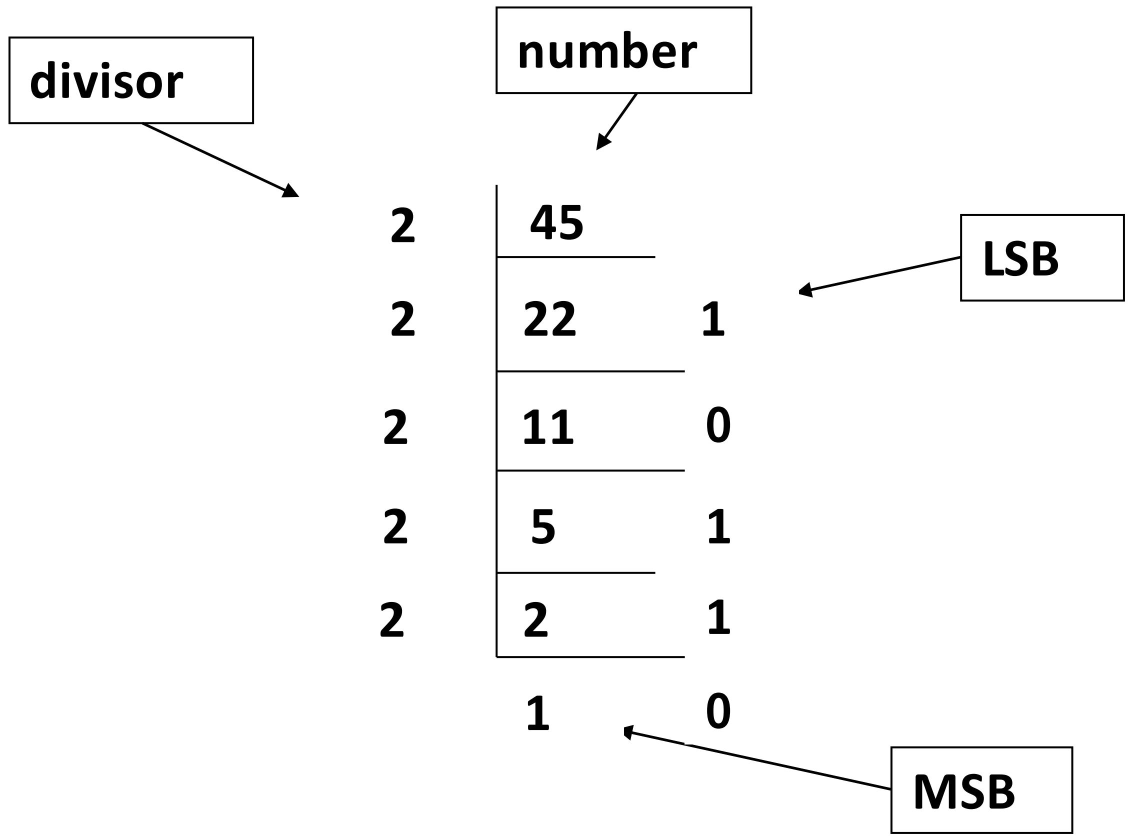
- $(10010.011)_2$
  $= 1x2^4 + 0x2^3 + 0x2^2 + 1x2^1 + 0x2^0 + 0x2^{-1} + 1x2^{-2} + 1x2^{-3}$
  $= 16 + 0 + 0 + 2 + 0 + 0 + 0.25 + 0.125 = (18.375)_{10}$

- $(111011.101)_2$
  $= 1x2^5 + 1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 0x2^{-2} +$
  $1x2^{-3} = 32 + 16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$
  $= (59.625)_{10}$

Decimal to Binary Conversion

- The method of converting a decimal number to binary involves dividing the number by 2, then dividing the resulting quotient by 2, then dividing the resultant quotient by 2, and so on.
- This continues till we encounter a number *i.e.*, 1, which is less than the divisor 2.
- The sequence of remainders obtained from these divisions is the binary equivalent of the decimal number, where the first remainder obtained is the ***least significant bit*** while the last remainder is the ***most significant bit***.

$$(45)_{10} = (101101)_2$$



- To convert a decimal fraction to binary, we first multiply the fraction by 2.
- The resulting product is a decimal number whose integer part is either 0 or 1.
- That integer is the first (most significant) bit of the binary equivalent.
- The fractional part of the product is then multiplied by 2, producing another decimal number whose integer part is either 0 or 1.
- This integer is the second bit of the binary equivalent.
- The above process continues, indefinitely, unless we reach a step where we are multiplying a zero fractional part by 2.
- In that case, all subsequent multiplication will produce 0’s, corresponding to non significant trailing 0’s, so as to terminate the conversion.

$(0.6875)_{10} = (0.1011)_2$

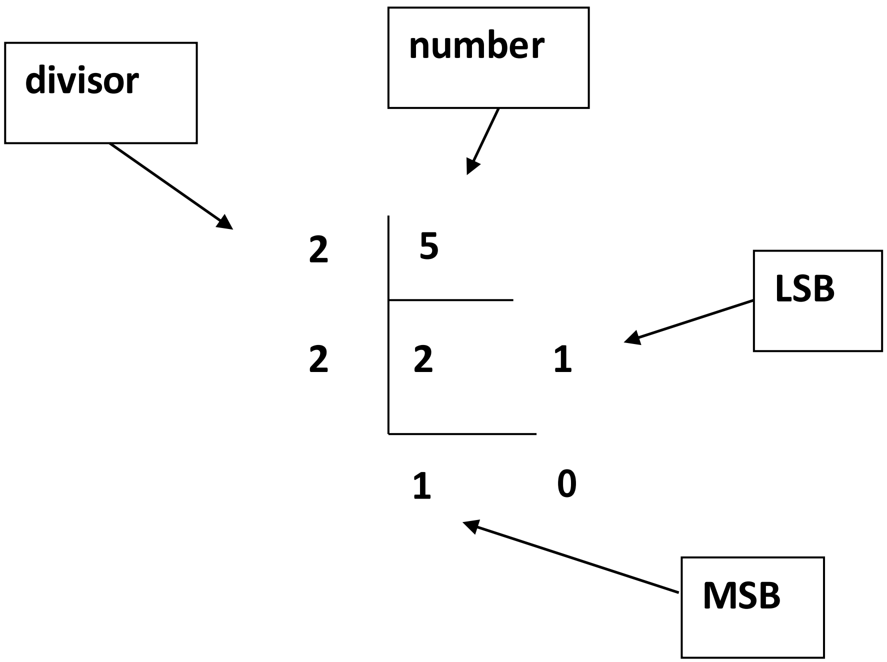
MSB

- 1^{st} step => 2x0.6875 = 1.375
- 2^{nd} step => 2x0.375 = 0.750
- 3^{rd} step => 2x0.750 = 1.500
- 4^{th} step => 2x0.500 = 1.000

LSB

It is important to note that the process terminated when we multiplied 2x0.500=1.000,
because all subsequent multiplications of 0.000 will produce 0.000

- To convert a decimal number having both an integer and a fractional part to its binary equivalent, the process involves conversion of each part separately.



To convert the fractional part, the multiplication process is used

$$2 \times 0.50 = 1.00$$
$$2 \times 1.00 = 0.00$$

$(5)_{10} = (101)_2$ $(0.5)_{10} = (0.1)_2$ $(5.5)_{10} = (101.1)_2$

THANK YOU